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The Quasar Luminosity Function as a Non-Homogeneous Poisson Process

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Abstract. The Quasar Luminosity Function (QLF) is the spatial density of quasars as a function of absolute optical magnitude M and redshift z. Following the work of Richards et al. (2006), this paper presents a flexible parametric model of quasars as a non-homogeneous poisson process over the $M \times z$ space. Our contribution to this model allows for the introduction of higher-order terms if warranted by the data. At present, M and z are treated as separable as is commonly done (Fan et al. 2001), however we plan to extend our model to account for the dependency apparent in the data. Parameters are estimated using the method of maximum likelihood. The performance of the Bayesian information criterion for model selection is examined on simulated data sets and a preliminary fitted model for SDSS quasar data is reported.

1. Introduction and Data

The quasar luminosity function (QLF) is a description of the number of quasars per Mpc^3 per mag as a function of K-corrected absolute optical magnitude M and redshift z. Estimation of luminosity functions is complicated by data truncation that occurs when objects are too faint or too far away to be detected.

The data is 14,113 quasars from the third edition of the Sloan Digital Sky Survey (SDSS) catalog. A major complicating factor is that quasars in this data set were not selected uniformly at random from the population of all detectable quasars in the universe. Rather, the selection probabilities for each quasar were dependent on that quasar's apparent brightness and its redshift distance from Earth. Richards et al. (2006) have obtained estimates of the sampling probabilities via simulation, and we will treat these probabilities as known.

2. A Parameterized Model for the Quasar Luminosity Function

The quasar luminosity function is modeled as a non-homogeneous poisson process (Lindsey 2004) in (M, z) space with an idealized intensity function of the form

$$\Phi_\theta(M, z) = 10^{\,\mu + f_1(M;\vec{\alpha}) + f_2(z;\vec{\beta})}\ , \text{ where}$$

$$f_1(M;\vec{\alpha}) = \sum_{i=1}^{A} \alpha_i \cdot (M - M_0)^i \quad , \text{and} \quad f_2(z;\vec{\beta}) = \sum_{j=1}^{B} \beta_j \cdot \log\left(\frac{1+z}{1+z_0}\right)^j$$

with model parameter $\theta = (\mu, \vec{\alpha}, \vec{\beta})$, $M_0 = -26$, and $z_0 = 2.45$. Complexity parameters A and B allow for the introduction of higher order terms if warranted by the data.

The maximum likelihood estimate for θ conditional on (A, B) can be obtained by maximizing

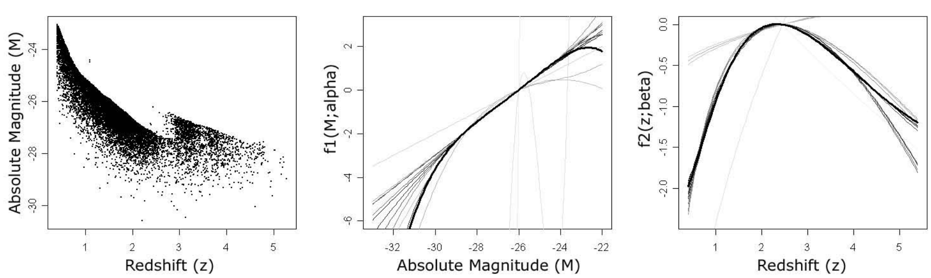
$$h(\theta|X) \;=\; -\int_z \int_M \Phi_\theta(M,z)\,\psi(M,z)\,\gamma(z)\,dM\,dz \;+\; \sum_{i=1}^{N} \log\left[\Phi_\theta(M_i,z_i)\right]$$

where $\psi(M, z)$ is the estimated sampling probability, $\gamma(z)$ is the infinitesimal volume differential measured in Mpc^3 of a spherical shell about the Earth at a given redshift z, and N is the sample size. Parameters for each model with $A \in \{1, 2, 3, 4\}$ and $B \in \{1, 2, 3, 4\}$ are estimated, and the final model is selected using the Bayesian information criterion $BIC(\hat{\theta}) = -2 \;\; l(\hat{\theta}|X) + k \log N$ (Schwarz 1978), where $l(\hat{\theta}|X)$ is the log-likelihood of the fitted model and $K = 1 + A + B$.

3. Simulations and Fitted Model for SDSS DR3 Quasar Data

The performance of this procedure was examined on simulated quasar data sets which were constructed according to predetermined parameterizations with fixed model complexities. Although the fitted models were accurate in estimating the poisson intensity surface, the Bayesian information criterion had a tendency to prefer overly complex models. We are currently working toward a more appropriate penalty term for use in the model selection procedure.

The first figure is a plot of the SDSS quasars by absolute magnitude M and redshift z. The last two figures show fitted curves $f_1(M; \vec{\alpha})$ and $f_2(z; \vec{\beta})$ under each complexity level. Darker lines represent higher preference for a given model under BIC. The selected model has $(A, B) = (4, 4)$ and is plotted in bold.



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